Critical Shear Loading of Curved Sandwich Panels Faced with Fiber-Reinforced Plastic

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The force-displacement relations of a cylindrically curved anisotropic sandwich panel faced with fiber-reinforced plastic (FRP) are derived by the complementary energy principle. The Rayleigh-Ritz method is then used for the buckling analysis of simply supported panels. The results show that 1) the quantitative difference between positive and negative shear buckling loadings; 2) panels of small aspect ratio are efficient to carry negative shear loads and those of large aspect ratio to carry marginally higher positive shear loads; 3) a fiber orientation angle of 50 deg measured from the curved edge provides maximum shear buckling strength; and 4) deeply curved shells resist higher negative shear loads and shallow shells higher positive shear loads.

Nomenclature

a = length of curved plane in x direction
A = extensional rigidity matrix of a face whose elements are A_{ij} $(i,j=1,2,3)$
c = thickness of core
$C_{mn}, D_{mn}, E_{mn}, F_{mn}, H_{mn}$ = undetermined coefficients of displacement functions [see Eq. (7)]
E_1, E_2 = elastic moduli of a layer in a face referred to its principal material directions 1 and 2
G_{12} = shear modulus of a layer in a face referred to its principal material directions 1 and 2
G_{xz}, G_{yz} = transverse shear moduli of core in xz and yz planes, respectively
h_k = normal distance of the inner surface of kth layer measured from xy plane
k_s = N_{xy}/A_{11}
m,n,p,q = half-wave length integers
N = number of layers in a face
N_{11}, N_{22} = resultants of internal forces per unit length in y and x directions, respectively
N_{12} = resultant of internal shear force per unit length in xy plane
Q_1, Q_2 = resultants of transverse shears, in xz and yz planes, per unit length
$\bar{Q}_{ij}^k$ $(i,j=1,2,3)$ = transferred reduced stiffness of kth layer
R_1, R_2, R_3, R_4, R_5 = A_{22}/A_{11}, A_{33}/A_{11}, A_{12}/A_{11}, A_{23}/A_{11}, A_{13}/A_{11}, respectively
R_6 = $(c+t)G_{yz}/A_{11}$
R_7 = $(c+t)G_{xz}/A_{11}$
R = radius of the cylindrical surfaces (midsurface)
t = thickness of face
u,v,w = midplane displacements of the shell along x, y, and z directions, respectively
α = angular measure in circumferential direction
α_m, β_n = $m\pi/a$, $n\pi/\gamma$, respectively
β_x, β_α = rotations, in xz and yz planes, of the normal to the midplane of the plate
γ = included angle of cylindrical edge
λ = $a/R\gamma$
θ = orientation of the fiber in a layer of the face measured from y direction
μ_{12} = Poisson's ratio referred to principal material directions

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Introduction

THE introduction of fiber-reinforced plastic (FRP) composites as a face material has further enhanced the efficiency of sandwich construction. The buckling behavior of curved sandwich plates with faces made of isotropic and specially orthotropic materials has received considerable attention in the literature. However, relatively little attention has been directed toward the buckling of FRP-faced, generally orthotropic (anisotropic) curved sandwich panels.

A small-deflection theory and subsequent analyses that take into account these deformations due to transverse shear were presented for the elastic behavior of orthotropic sandwich plates having a constant cylindrical curvature by Stein and Mayers.[1,3] The overall buckling and wrinkling of sandwich cylinders under compressive and torsional loads were studied by Gerard,[2] whose theoretical studies wee substantiated by experiments. By the method of complementary energy, Wang[4] derived stress-strain relations for isotropic sandwich plates and shells. Galerkin's method of buckling isotropic and corrugated sandwich cylinders under combined loading was studied, respectively, by Wang et al.[5] and Baker.[6] Specially orthotropic FRP-faced sandwich shells were studied for compressive buckling by Norris and Zahn,[7] for fabrication and full-scale structural evaluation by Nordby et al.,[8-10] and for vibration analysis by Bert and Ray.[11] A bending deflection analysis of cylindrical shells with laminated faces was presented by Schmit and Monforton.[12] Rao[13] presented a buckling analysis of FRP-faced flat rectangular generally orthotropic sandwich plates under combined loading. Theoretical and experimental studies on the buckling of flat sandwich panels with laminated faces under uniform compression were reported by Pearce and Webber.[14,15] Recently, Rao and Kaeser[16] presented the buckling analysis of a flat anisotropic sandwich panel under shear loading. Shear buckling of a simply supported rectangular sandwich panel with constant cylindrical curvature and having specially orthotropic facings and core was presented using Galerkin's method by Davenport and Bert.[17] Sayed[18] discussed the critical shear loading on curved panels of corrugated sheets.

While the buckling of specially orthotropic sandwich shells under either simple shear or combined loading has been the subject of limited investigation, shear buckling of generally orthotropic curved sandwich plate with FRP facings is not found in the literature. This paper is concerned with overall buckling of simply supported FRP-faced, generally orthotropic, cylindrically curved sandwich plates subjected to edge shear loading. The plate is built symmetrically with faces made by lamination. The reinforcing fibers in each layer are oriented at an arbitrary angle measured with respect to the cylindrical edge (Fig. 1). The faces are thin and can be thus treated as generally orthotropic membranes; the core is specially orthotropic.

Formulation

Reissner[19] formulated the stress-strain relations of isotropic sandwich shells by means of Castigliano's theorem of minimum complementary energy. Reissner's theory is extended in this paper to derive the force displacement relations of anisotropic curved sandwich plates; with the help of these relations, the Rayleigh-Ritz method is used to analyze a simply supported plate.

The final form of force-displacement relations is

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = 2 \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial v}{R\partial\alpha} + \dfrac{w}{R} \\ \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{R\partial\alpha} \end{Bmatrix} \tag{1}$$

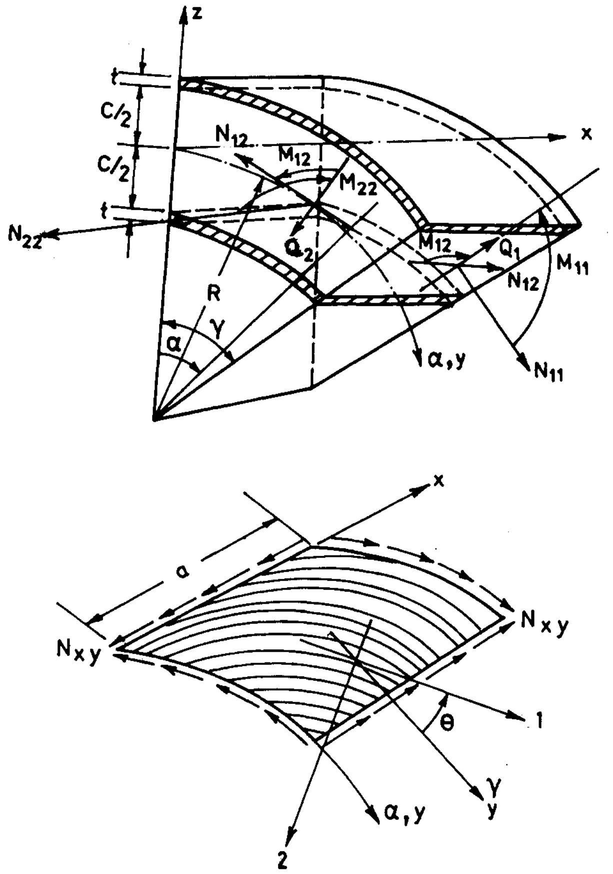

Fig. 1 Geometry and loading.

$$\begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \frac{(c+t)^2}{2} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \beta_\alpha}{R\partial\alpha} \\ \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{\partial \beta_\alpha}{\partial x} + \dfrac{\partial \beta_x}{R\partial\alpha} \end{Bmatrix} \tag{2}$$

where u, v, and w are the displacement components of the midsurface along the x, α, and z axes; β_x and β_α the rotations of the normal to the middle surface in xz and yz planes, respectively; c the thickness of core; t the thickness of each face; N_{11}, N_{12}, and N_{22} are the stress resultants; M_{11}, M_{12}, and M_{22} the stress couples (Fig. 1); α the angular measure in circumferential direction; and A_{ij} the extensional rigidities of each face. For the derivation of Eqs. (1) and (2), it is assumed that the shell wall thickness is negligibly small compared to the radius of curvature R of the middle surface. The thin faces are characterized by effective stress-strain relations[20] and the extensional rigidities A_{ij} of them are given by

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}^{k} (h_k - h_{k-1}) \tag{3}$$

where N is the number of layers in each face, h_k the normal distance of the inner surface of the kth layer measured from xy plane (Fig. 1), and $\bar{Q}_{ij}^{k}$ the transformed reduced stiffnesses referred to geometric axes given in terms of fiber orientation angle θ (Fig. 1) and principal material properties of each layer.[20] The transverse shear forces Q_1 and Q_2 (Fig. 1) are

$$Q_1 = (c+t)G_{yz}\left(\beta_\alpha + \frac{\partial w}{R\partial\alpha} - \frac{v}{R}\right)$$

$$Q_2 = (c+t)G_{xz}\left(\beta_x + \frac{\partial w}{\partial x}\right) \tag{4}$$

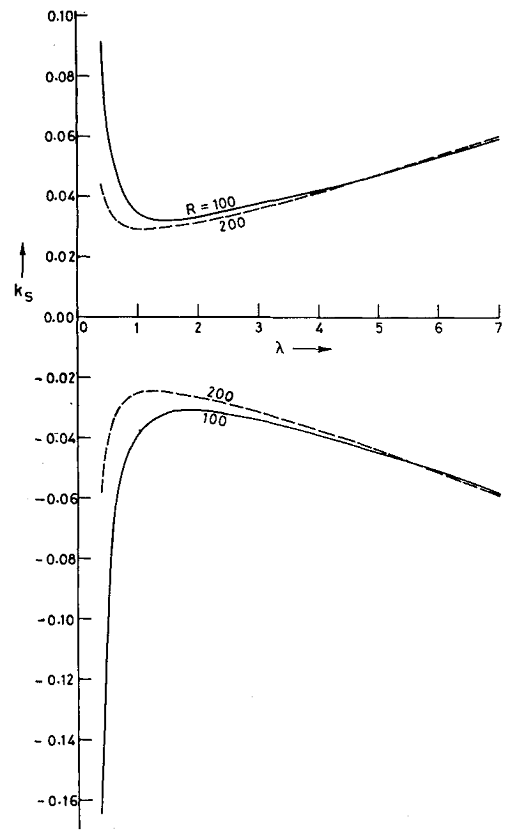

Fig. 2 Variation of k_s with λ ($\theta = 45$ deg and $c/t = 20$).

where G_{xz} and G_{yz} are the transverse shear moduli of the core in the xz and yz planes, respectively. For the derivation of force-displacement relations [Eqs. (1), (2), and (4) of a sandwich shell with thin faces], it is assumed that the core thickness is extended to the middle surface of each face.

The potential energy V needed for the Rayleigh-Ritz method is given by

$$\begin{aligned} V=\frac{1}{2}\int_x\int_\alpha \Big\{ & 2A_{11}\left(\frac{\partial v}{R\partial\alpha}+\frac{w}{R}\right)^2+2A_{22}\left(\frac{\partial u}{\partial x}\right)^2 \\ & +2A_{33}\left(\frac{\partial u}{R\partial\alpha}+\frac{\partial v}{\partial x}\right)^2+4A_{12}\left(\frac{\partial v}{R\partial\alpha}+\frac{w}{R}\right)\frac{\partial u}{\partial x} \\ & +4A_{23}\left(\frac{\partial u}{R\partial\alpha}+\frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial x} \\ & +4A_{13}\left(\frac{\partial v}{R\partial\alpha}+\frac{w}{R}\right)\left(\frac{\partial u}{R\partial\alpha}+\frac{\partial v}{\partial x}\right) \\ & +(c+t)^2\left[A_{11}\left(\frac{\partial\beta_\alpha}{R\partial\alpha}\right)^2\Big/2+A_{22}\left(\frac{\partial\beta_x}{\partial x}\right)^2\Big/2\right. \\ & +A_{33}\left(\frac{\partial\beta_x}{R\partial\alpha}+\frac{\partial\beta_\alpha}{\partial x}\right)^2\Big/2 \\ & +A_{12}\frac{\partial\beta_\alpha}{R\partial\alpha}\frac{\partial\beta_x}{\partial x}+A_{23}\frac{\partial\beta_x}{\partial x}\left(\frac{\partial\beta_x}{R\partial\alpha}+\frac{\partial\beta_\alpha}{\partial x}\right) \\ & \left.+A_{13}\frac{\partial\beta_\alpha}{R\partial\alpha}\left(\frac{\partial\beta_x}{R\partial\alpha}+\frac{\partial\beta_\alpha}{\partial x}\right)\right] \\ & +(c+t)\left[G_{yz}\left(\beta_\alpha+\frac{\partial w}{R\partial\alpha}-\frac{v}{R}\right)^2\right. \\ & \left.+G_{xz}\left(\beta x+\frac{\partial w}{\partial x}\right)^2\right]-2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{R\partial\alpha}\Big\}R\mathrm{d}\alpha\mathrm{d}x \end{aligned} \tag{5}$$

where N_{xy} is the external shear force distributed uniformly along the edges (Fig. 1).

The plate is simply supported along the edges. The edges are also reinforced to prevent the displacement in the plane of the cross sections. The boundary conditions of such a plate are idealized by

$$v=w=\beta\alpha=N_{22}=M_{22}=0 \text{ at } x=0 \text{ and } a$$

$$u=w=\beta x=N_{11}=M_{11}=0 \text{ at } \alpha=0 \text{ and } \gamma \tag{6}$$

where a is the length of straight edge and γ the included angle of the curved edge.

Analysis

The problem is analyzed by the Rayleigh-Ritz method that needs the field variables u, v, w, β_x, and β_α in explicit form. The trial functions for these variables are selected to satisfy at least the geometric boundary conditions of Eq. (6). If they also satisfy the natural boundary conditions, the accuracy and convergence of the solution improves. In practice, satisfying the natural boundary conditions (fully or partially) is difficult, but not impossible. The trial functions used in the present investigation are

$$u=\sum_m\sum_n C_{mn}\cos\alpha_m\times\sin\beta_n\alpha$$

$$v=\sum_m\sum_n D_{mn}\sin\alpha_m\times\cos\beta_n\alpha$$

$$w=\sum_m\sum_n E_{mn}\sin\alpha_m\times\sin\beta_n\alpha$$

$$\beta_x=\sum_m\sum_n F_{mn}\cos\alpha_m\times\sin\beta_n\alpha$$

$$\beta_\alpha=\sum_m\sum_n H_{mn}\sin\alpha_m\times\cos\beta_n\alpha \tag{7}$$

where $\alpha_m=m\pi/a$ and $\beta_n=n\pi/\gamma$; m and n are half-wavelength integers; and C_{mn}, D_{mn}, E_{mn}, F_{mn}, and H_{mn} are the undetermined coefficients. The trial functions of Eq. (7) fully satisfy the geometric boundary conditions. But because of the presence of stretching-shearing coupling stiffness terms A_{13} and A_{23} in Eqs. (1) and (2), the natural boundary conditions (i.e., $N_{22}=M_{22}=0$ at $x=0$ and a and $N_{11}=M_{11}=0$ at $\alpha=0$ and γ) are not satisfied fully. The resulting inaccuracy in the solution can be compensated by retaining a sufficiently large number of terms in the series of Eq. (7).

Minimizing the potential energy V of Eq. (5), after substituting the trial solution of Eq. (7), integrating over the entire area of the plate, and using the notation

$$\lambda=a/R\gamma \qquad R_1=A_{22}/A_{11} \qquad R_2=A_{33}/A_{11}$$

$$R_3=A_{12}/A_{11} \qquad R_4=A_{23}/A_{11} \qquad R_5=A_{13}/A_{11}$$

$$R_6=(c+t)G_{yz}/A_{11} \qquad R_7=(c+t)G_{xz}/A_{11}$$

$$k_s=N_{xy}/A_{11} \tag{8}$$

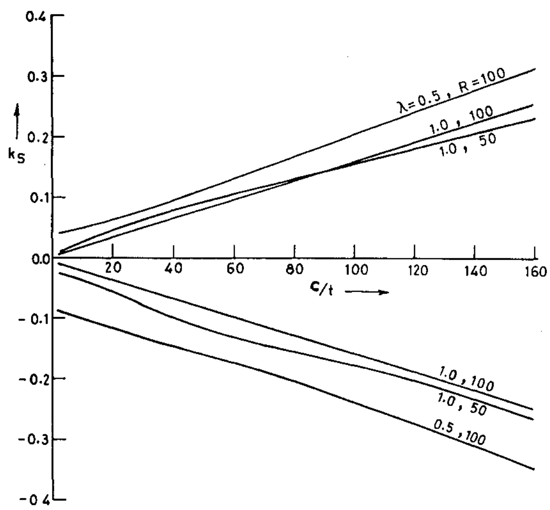

Fig. 3 Variation of k_s with c/t ($\theta=45$ deg).

Table 1 Results of comparison with Ref. 13 ($c/t=20$ and $\theta=45$ deg)

		λ						
	Investigation	0.4	0.6	0.8	1.0	2.0	3.0	5.0
$+N_{xy}$	Ref. 13	77.53	78.51	80.23	83.14	100.85	118.02	157.29
	Present	79.49	80.79	82.83	86.08	105.19	123.39	165.29
$-N_{xy}$	Ref. 13	43.58	45.37	48.28	52.73	79.37	101.97	148.19
	Present	44.15	46.07	49.17	53.86	82.03	105.97	154.77

one obtains a set of five recurring algebraic equations in the five unknowns (C_{mn}, D_{mn}, E_{mn}, F_{mn}, and H_{mn}) as follows.

$$\frac{\partial V}{\partial C_{mn}}=0:$$

$$4\left(R_1\alpha_m^2+\frac{R_2\beta n^2}{R^2}\right)C_{mn}+\frac{4(R_2+R_3)\alpha_m\beta_n D_{mn}}{R}-\frac{4R_3\alpha_m E_{mn}}{R}-64R_4\sum_p\sum_q\frac{(\alpha_m\beta_q mn+\alpha_p\beta_n\cdot pq)C_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}-64\sum_p\sum_q\frac{(R_5\beta_n\beta_q pq/R^2+R_4\alpha_p\alpha_m mn)D_{pq}}{\pi^2(m^2-p^2)(n^2-q^2)}+64R_5\sum_p\sum_q\frac{\beta_n pqE_{pq}}{\pi^2R^2(m^2-p^2)(n^2-q^2)}=0 \quad (9)$$

$$\frac{\partial V}{\partial D_{mn}}=0:$$

$$\frac{4\alpha_m\beta_n(R_2+R_3)C_{mn}}{R}+\left(\frac{4\beta_n^2}{R^2}+4R_2\alpha_m^2+\frac{2R_6}{R^2}\right)D_{mn}-(4+2R_6)\frac{\beta_n E_{mn}}{R^2}-2R_6\frac{H_{mn}}{R}-64\sum_p\sum_q\frac{(R_4\alpha_m\alpha_p pq+R_5\beta_n\beta_q mn/R^2)C_{pq}}{\pi^2(m^2-p^2)(n^2-q^2)}-64R_5\sum_p\sum_q\frac{(\alpha_p\beta_n mn+\alpha_m\beta_q pq)D_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}+64R_5\sum_p\sum_q\frac{\alpha_m pqE_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}=0 \quad (10)$$

$$\frac{\partial V}{\partial E_{mn}}=0:$$

$$-4\frac{R_3\alpha_m C_{mn}}{R}-(4+2R_6)\frac{\beta_n D_{mn}}{R^2}+\left(\frac{4}{R^2}+\frac{2R_6\beta_n^2}{R^2}+2R_7\alpha_m^2\right)E_{mn}+2R_7\alpha_m F_{mn}+\frac{2R_6\beta_n H_{mn}}{R}+64R_5\sum_p\sum_q\frac{\beta_q mnC_{pq}}{\pi^2R^2(m^2-p^2)(n^2-q^2)}+64R_5\sum_p\sum_q\frac{\alpha_p mnD_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}+64k_s\sum_p\sum_q\frac{\alpha_m\beta_q npE_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}=0 \quad (11)$$

$$\frac{\partial V}{\partial F_{mn}}=0:$$

$$2R_7\alpha_m E_{mn}+\left[(c+t)^2R_1\alpha_m^2+\frac{(c+t)^2R_2\beta_n^2}{R^2}+2R_7\right]F_{mn}+\frac{(c+t)^2(R_2+R_3)\alpha_m\beta_n H_{mn}}{R}-16(c+t)^2R_4\sum_p\sum_q\frac{(\alpha_m\beta_q mn+\alpha_p\beta_n pq)F_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}-16(c+t)^2\sum_p\sum_q\frac{(R_4\alpha_m\alpha_p mn+R_5\beta_n pq/R^2)H_{pq}}{\pi^2(m^2-p^2)(n^2-q^2)}=0 \quad (12)$$

$$\frac{\partial V}{\partial H_{mn}}=0:$$

$$-\frac{2R_6 D_{mn}}{R}+\frac{2R_6\beta_n E_{mn}}{R}+\frac{(c+t)^2(R_2+R_3)\alpha_m\beta_n F_{mn}}{R}+\left[(c+t)^2\left(\frac{\beta_n^2}{R^2}+R_2\alpha_m^2\right)+2R_6\right]H_{mn}-16(c+t)^2\sum_p\sum_q\frac{(R_4\alpha_m\alpha_p pq+R_5\beta_n\beta_q mn/R^2)F_{pq}}{\pi^2(m^2-p^2)(n^2-q^2)}-16(c+t)^2R_5\sum_p\sum_q\frac{(\alpha_p\beta_n mn+\alpha_m\beta_p pq)H_{pq}}{\pi^2R(m^2-p^2)(n^2-q^2)}=0 \quad (13)$$

Equations (9-13) are homogeneous and algebraic. For the nontrivial solution of the unknowns C_{mn}, D_{mn}, E_{mn}, F_{mn}, and H_{mn}, the determinant of the coefficients of Eqs. (9-13) must be zero. This condition forms the criterion for the determination of shear buckling coefficient k_s of Eq. (11).

Computations

Equations (9), (10), (12), and (13) are solved for C_{mn}, D_{mn}, F_{mn}, and H_{mn} in terms of E_{mn} and are then substituted in Eq. (11). The algebraic equations resulting from this process can be cast into a matrix form as

$$[P]E=k_s[Q]E \quad (14)$$

where P and Q are square matrices and E the column matrix of the coefficients E_{mn}. Equation (14) is in the form of a general eigenvalue problem that can be solved by a standard eigenvalue subroutine. IMSL subroutines and the Cyber 174 computer at the Swiss Federal Institute of Technology in Zurich are used. For the purpose of evaluation of numerical results, the first 16 terms of the series of Eq. (7) are retained, i.e., $m,n=1,2,3,4$.

Table 2 Extensional rigidities, A_{ij}

θ, deg	$A_{11}\times10^{-4}$	R_1	R_2	R_3	R_4	R_5
0	0.78220	0.22441	0.09875	0.07495	0.0	0.0
10	0.74836	0.23823	0.12398	0.09911	0.01225	0.12638
20	0.65633	0.29192	0.20133	0.17298	0.04884	0.24823
30	0.53087	0.42862	0.33322	0.29817	0.13903	0.35580
40	0.40266	0.73838	0.51187	0.46565	0.31450	0.42737
45	0.34598	1.00000	0.60731	0.55352	0.43836	0.43836
60	0.22754	2.33310	0.77744	0.69566	0.83012	0.32437
70	0.19160	3.42557	0.68968	0.59255	0.85033	0.16932
80	0.17828	4.19760	0.52042	0.41604	0.53050	0.05141
90	0.17553	4.45610	0.44002	0.33400	0.0	0.0

Numerical Comparisons

The analysis of the present investigation is checked by comparing the result with those of Davenport and Bert,[17] and Rao.[13] The shear buckling load N_{xy} evaluated in Ref. 17 is 630 lb/in. and of others reported there 685 lb/in. With the same data, N_{xy} given by the present investigation is 731 lb/in.

Rao[13] presented buckling analysis of a simply supported FRP-faced anisotropic flat sandwich plate. The data used for comparison (any consistent units can be used) are

$$t=0.315, \quad a=225, \quad N=1, \quad \theta=45 \text{ deg}, \quad c/t=20.0$$

the core properties are

$$G_{xz}=21.0 \text{ and } G_{yz}=10.5$$

and the face properties

$$E_1=24210.0, \quad E_2=5433.0, \quad \mu_{12}=0.334, \quad G_{12}=2452.0$$

To reduce the curved plate formulation of the present work to represent the flat plate of Ref. 13, a sufficiently large radius is taken, the value being $R=100{,}000.0$. The results are compared in Table 1. The present solution is very close to that of Rao.[13] From the above two comparisons, one can conclude the validity of the formulation, analysis, and computation in the present work.

Numerical Results

Each face is assumed to be made of a typical glass fiber-reinforced composite, with a core of hexagonal-cell honeycomb structures. The principal material properties of face layers and the transverse shear moduli of the core are those used for comparison with Ref. 13. With the intention of emphasizing the influence of anisotropy on the buckling behavior of the plate, each face is assumed to be built of a single FRP layer. The shear buckling coefficient k_s is evaluated and presented in Figs. 2-5, varying such parameters as the aspect ratio λ, fiber orientation angle θ, core-to-face thickness ratio c/t, and radius R of the cylindrical surface. Table 2 provides the extensional rigidities of a face sheet for the calculation of shear load N_{xy} from the coefficient k_s.

Discussion

The nondimensionalization scheme of Eq. (8) is different from the conventional one[13]; according to the present scheme, which contains only the material parameter A_{11}, the trend of variation of N_{xy} is the same as that of k_s provided the extensional rigidities A_{ij} remain constant over the range of parameter of interest.

It is clear from Figs. 2-5 that the positive and negative shear buckling loads of anisotropic sandwich plates are different, the sign convention for positive shear load being as shown in Fig. 1, whereas for the case of isotropic and specially orthotropic plates ($\theta=0$ and 90 deg in Fig. 4), there is no difference between positive and negative shear buckling strengths.

Influence of Aspect Ratio

Figure 2 illustrates the effect of aspect ratio λ on the shear buckling coefficients k_s. The numerical values of k_s decrease with λ up to $\lambda=1.5$; for all larger values of λ, k_s increases continuously. The plates with small aspect ratios ($\lambda<1.0$) possess higher negative than positive shear buckling strength. For $1<\lambda<3$, the positive shear buckling load is marginally higher than for the negative one. For $\lambda>3$, the positive and negative values of k_s are close to each other. Hence, for the plate to possess a higher shear buckling load, it is advisable to choose the aspect ratio according to the nature of the shear loading.

Influence of Core-to-Face Thickness Ratio (c/t)

Both positive and negative shear buckling coefficients increase (Fig. 3) with core-to-face thickness ratio c/t. Over the range $0<c/t<160$ considered, the plate is more stable under negative shear at either low aspect ratio or small radius of cylindrical surface (say, $\lambda=0.5$ and $R=100$ or $\lambda=1$ and

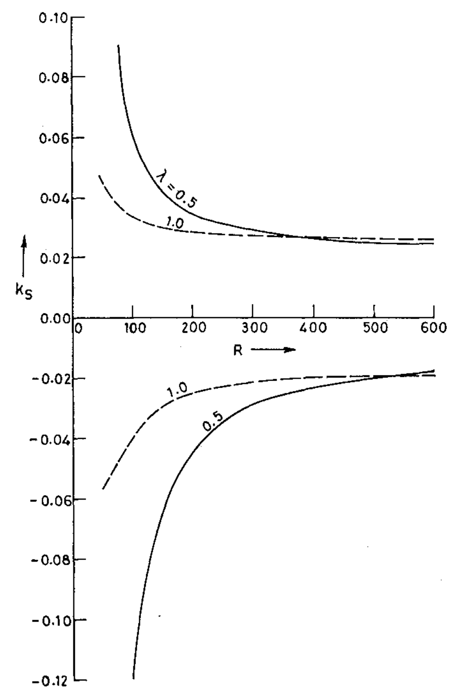

Fig. 4 Variation of k_s with R ($\theta=45$ deg and $c/t=20$).

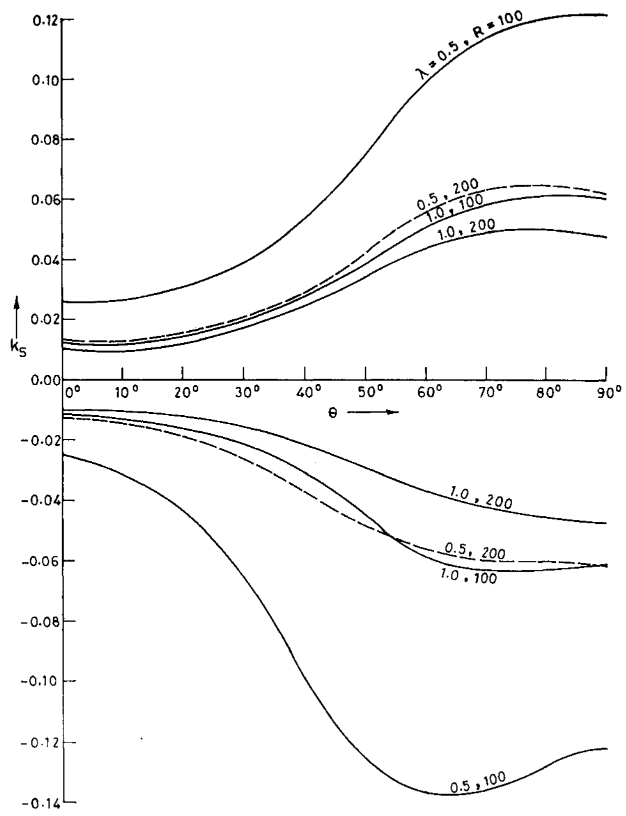

Fig. 5 Variation of k_s with θ ($c/t=20$).

$R=50$; see Fig. 3). At a combination of large aspect ratio and large radius (say, $\lambda=1$ and $R=100$), the positive and negative values of buckling coefficient k_s are almost equal.

Influence of Fiber Orientation Angle θ

The shear buckling coefficient k_s increases nonlinearly with θ up to about 65 deg (Fig. 5). Over $65<\theta<90$ deg, k_s decreases. When the aspect ratio λ is small (say, $\lambda=0.5$, Fig. 5), the positive shear coefficient is smaller than the negative for $0<\theta<90$ deg. But, the plates of higher aspect ratio and radius (say, $\lambda=1$ and $R=200$, Fig. 5) possess positive shear strength higher than the negative for the values of θ lying between 40 and 70 deg. Depending upon the aspect ratio, the radius R, and the nature of shear, the value of the fiber orientation angle θ at which shear buckling load N_{xy} is maximum varies over a range of 40-60 deg. As a rule of thumb, it is recommended that the fibers be oriented at an angle of 50 deg measured from the curved edge.

Influence of Radius of Curvature R

Figure 4 indicates the nature of variation of shear buckling coefficient k_s with radius of curvature R of the cylindrical surface. The variation is quite nonlinear over $0<R<300$. For $R>300$, the variation either becomes linear ($\lambda=0.5$ of Fig. 4) or negligible ($\lambda=1$ of Fig. 4). For deeply curved shells, the positive shear stability is smaller than the negative. For shells of low curvature, the reverse is true.

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